Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 21-054
MAS-S2
2022/2164
Web version of 2021/61813

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	35
Section Two: Calculator-assumed	11	11	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2021: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (92 Marks)**

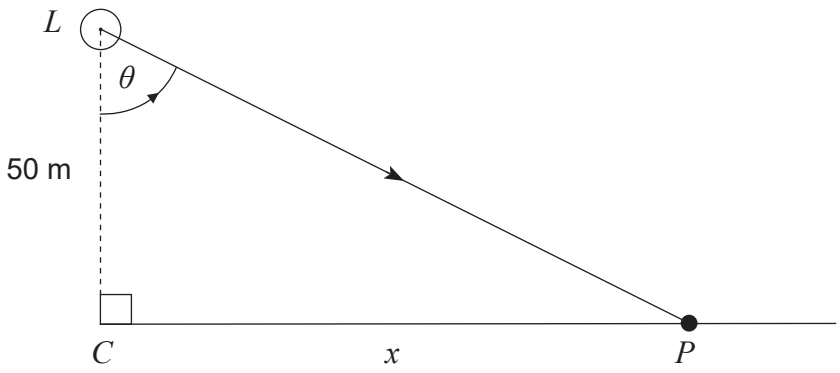
This section has **11** questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

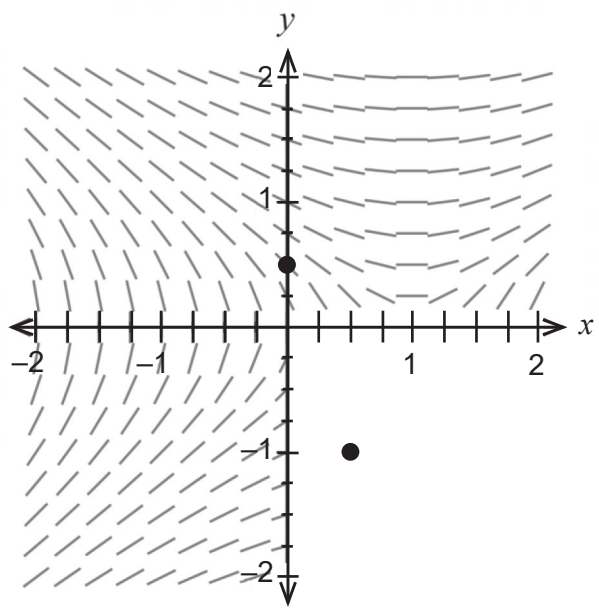
Question 9 **(5 marks)**

A beam of light completes three revolutions each minute from a lighthouse L that is 50 metres from a coastline. Determine the speed of the beam of light moving along the coast when it is at point P, 100 metres up the coast, correct to the nearest 0.01 metres per second.



Question 10 **(8 marks)**

Part of the slope field given by $\frac{dy}{dx} = \frac{x-1}{2y}$ is shown below.



(a) Calculate and draw the slope field at the point $(0.5, -1)$. (3 marks)

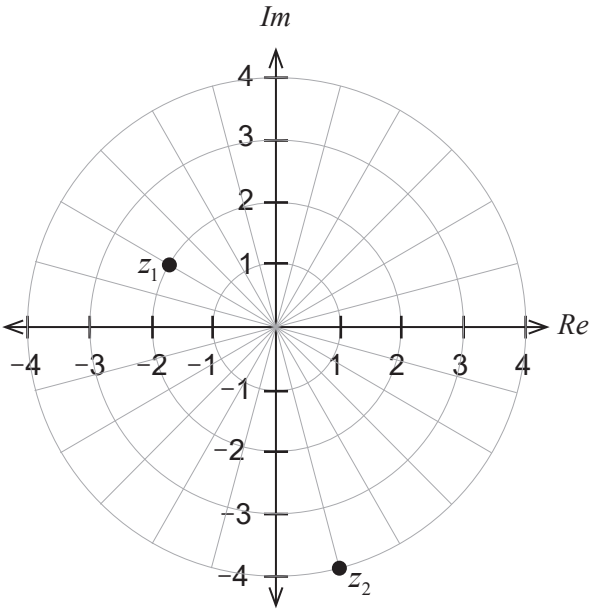
(b) Determine the equation of the solution curve that contains the point $(0, 0.5)$. (3 marks)

(c) Draw the solution curve that contains the point $(0, 0.5)$. (2 marks)

See next page

Question 11 **(9 marks)**

Two complex numbers z_1 and z_2 are shown in the Argand plane below.



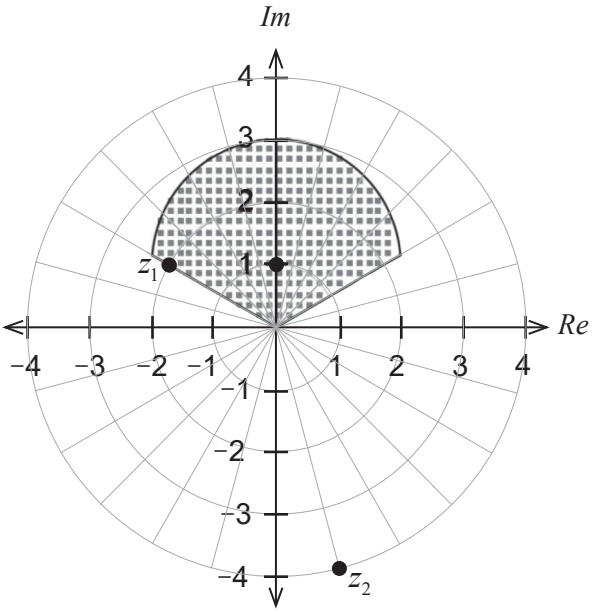
(a) Write the expression for z_1 in exact polar form. (2 marks)

(b) Write the expression for z_1 in exact Cartesian form. (1 mark)

(c) Plot the complex number iz_2 on the Argand diagram above. (2 marks)

See next page

(d) A sketch of the locus of a complex number z is shown below. The upper boundary of the locus is part of a circle, centred at $z = i$. Write equations or inequalities in terms of z (without using $x = Re(z)$ or $y = Im(z)$) for the indicated locus. (4 marks)



Question 12 **(6 marks)**

The horizontal displacement of a Ferris wheel cabin exhibits simple harmonic motion. The maximum horizontal speed is $\frac{\pi}{2}$ metres per second and its period of motion is exactly 60 seconds.

Let $x(t) = A\cos(nt)$ be the horizontal displacement after t seconds.

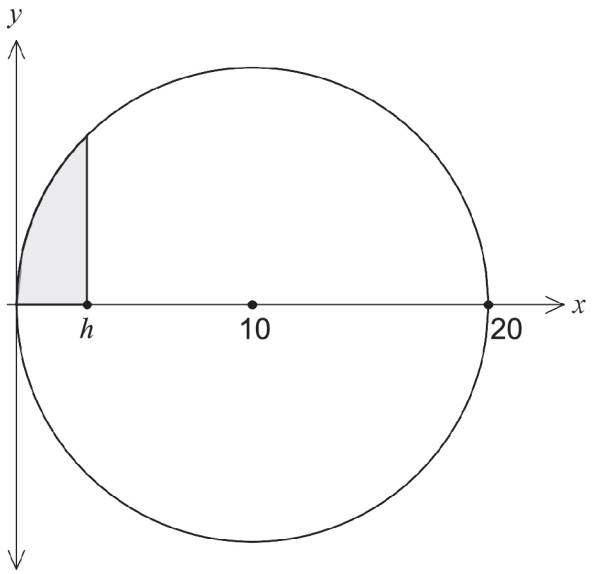
(a) Determine the values of A and n. (3 marks)

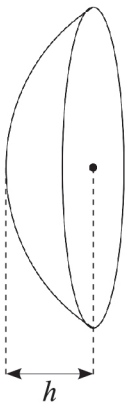
(b) Determine the horizontal acceleration, correct to the nearest 0.001 m/s^2, when the horizontal displacement is 10 metres. (3 marks)

See next page

Question 13 **(5 marks)**

A solid spherical cap with depth h is part of a solid sphere with radius 10 cm. This cap can be generated by revolving the shaded region about the x axis.





(a) Show that the equation for the circle shown above is $x^2 + y^2 = 20x$. (1 mark)

(b) Develop an expression for the volume of the spherical cap in terms of h. (4 marks)

See next page

Question 14 **(10 marks)**

On a Saturday afternoon, three separate family groups visit their local cinema to watch a feature movie. The cinema names this as DollarDay where the ticket prices for adults, children and pensioners are charged in whole dollar amounts.

The table below indicates the number of people in each category and the total paid for each family group.

Group	Adults	Children	Pensioners	Total cost
1	2	4	–	$108
2	3	6	–	$162
3	2	5	2	$152

Let a = the price for each adult ($)
c = the price for each child ($)
p = the price for each pensioner ($)

(a) Formulate the equations that can be used to determine the ticket prices. (1 mark)

(b) Using the equations formed, determine the total cost for a group consisting of 1 child accompanied by 2 pensioners. (2 marks)

(c) Solve simultaneously the equations formulated in part (a). (2 marks)

See next page

(d) Explain the geometric interpretation of the equations and the simultaneous solution. (2 marks)

Now assume that the price for an adult is greater than the price for a child and that the price for a pensioner is the lowest priced ticket.

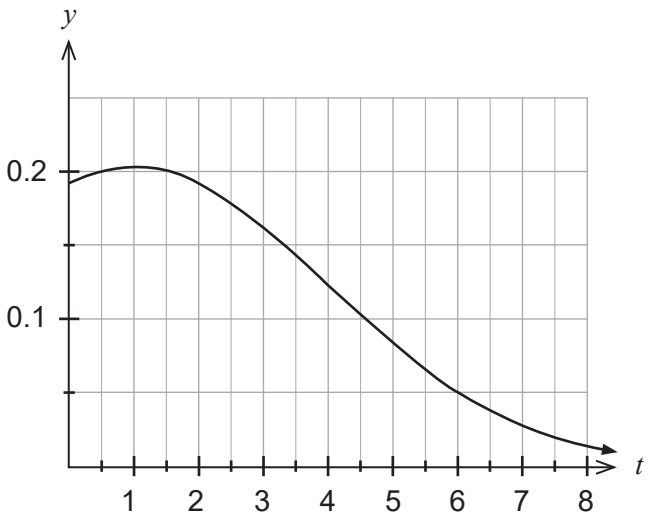
(e) Determine the ticket prices for adults, children and pensioners on DollarDay. (3 marks)

Question 15 **(9 marks)**

An experiment was conducted to measure how quickly adults respond to the request: 'send me a text message'.

Let T = the number of hours taken for an adult to respond and send a text message.

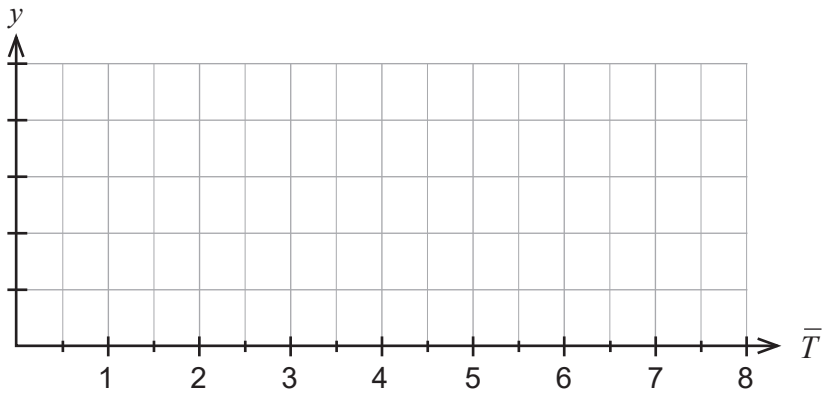
It was found that the distribution of the population of response times for adults was given by the probability density function shown below, with mean $\mu = 3$ hours and standard deviation $\sigma = 2.4$ hours.



Random samples of size 64 are drawn repeatedly from the population of response times and the sample mean response time $\overline{T}$ is determined for each sample.

(a) Calculate, correct to 0.001, the probability that a sample mean response time will be between 150 minutes and 210 minutes. (3 marks)

(b) Sketch the likely distribution of the sample mean $\overline{T}$ (for samples of size 64) on the axes below. (2 marks)



Anika, a teacher at the TekNoCrat School, theorises that as teenagers tend to check their text messages more frequently than adults, then the population mean response time for teenagers will be much lower than the population mean adult response time $\mu = 3$.

Anika is then presented with the sample mean response time for a sample gathered from an unknown source.

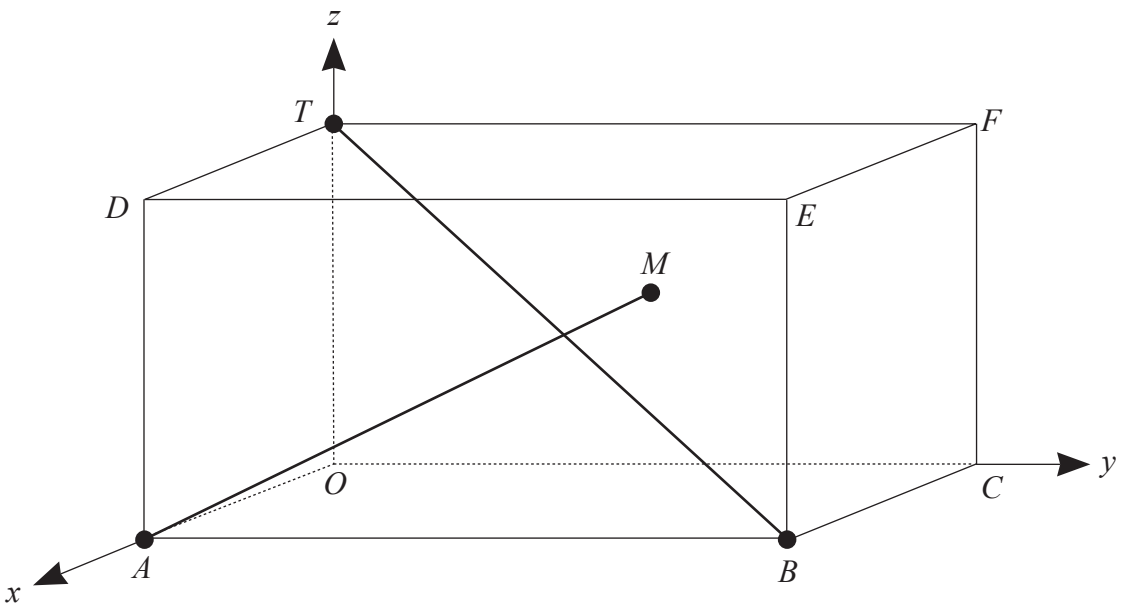
Sample size	Sample mean (hours)	Sample standard deviation (hours)
100	2.1	2.7

Calculations are performed and Anika concludes by stating: 'this sample was clearly not taken from the population of adult response times. It is highly likely that this sample was taken from a sample of 100 teenagers'.

(c) Perform the necessary calculations and comment on Anika's claim. (4 marks)

Question 16 **(8 marks)**

A rectangular prism is defined using the coordinate system shown with $A\left(2, 0, 0\right)$, $C\left(0, 4, 0\right)$ and $T\left(0, 0, 3\right)$. Point M is the centre of the planar face $OCFT$ with coordinates $\left(0, 2, 1.5\right)$.



(a) Determine the vector equation for the prism's main diagonal $\overleftrightarrow{BT}$. (2 marks)

(b) Determine the Cartesian equation of the sphere that contains all vertices of the rectangular prism. (3 marks)

(c) Prove, using a vector method, that line $\overleftrightarrow{AM}$ does **not** intersect $\overleftrightarrow{BT}$. (3 marks)

See next page

Question 17 **(12 marks)**

A researcher is interested in estimating the population mean μ (dollars) that Perth residents had spent via online shopping in December 2020. A random sample of size n gave a sample mean of \$400, a sample standard deviation s and a 95% confidence interval of width \$200.

(a) State the 95% confidence interval obtained. (1 mark)

(b) Calculate the standard deviation of the sample mean, correct to \$0.01. (2 marks)

(c) In terms of n, what sample size would yield a 95% confidence interval of width \$50? Show your reasoning. (2 marks)

(d) What is the probability that another sample of size $2n$ would produce a sample mean that differs from μ by more than \$50? (3 marks)

Four different confidence intervals (A, B, C and D) are obtained for the mean amount spent via online shopping by Perth residents in December 2020.

Confidence interval	Sample size	Sample standard deviation	Confidence level
A	n	s	95%
B	n	s	99%
C	$2n$	s	95%
D	n	$0.8s$	95%

(e) Which of the confidence intervals (A, B, C or D) contains μ, the population mean expenditure for online shopping in December 2020? Justify your answer. (2 marks)

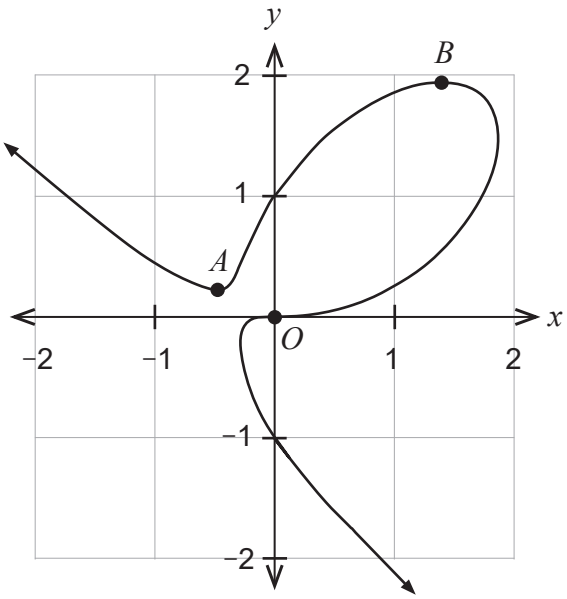
(f) For each of the following, state the confidence interval that has the smaller width. Justify your answers.

(i) A and B. (1 mark)

(ii) C and D. (1 mark)

Question 18 **(6 marks)**

The equation $x^3 + y^3 = 3xy + y$ implicitly defines the curve shown below.



(a) Using implicit differentiation obtain the expression for $\frac{dy}{dx}$. (3 marks)

The slope of the curve at the origin O and points A and B is equal to zero.

(b) Show that the equation that determines the x coordinates for points A and B is given by $x^4 - 2x - 1 = 0$ and hence determine the coordinates for point A correct to 0.001.

(3 marks)

See next page

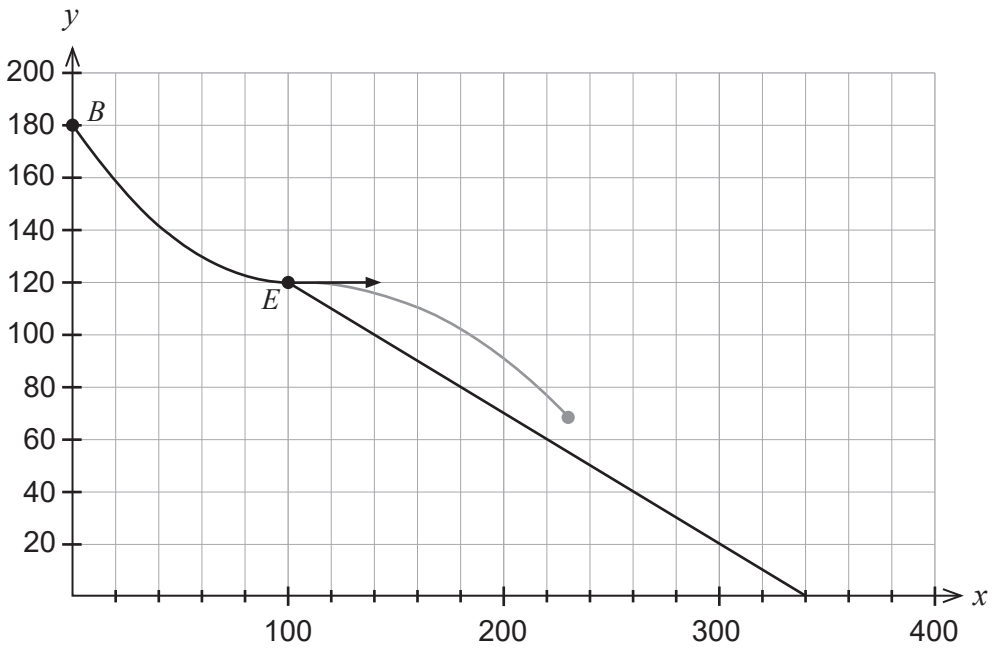
Question 19 **(14 marks)**

Using the correct technique, Olympic ski jumpers can slow down their descent, by creating lift to counteract gravity. These jumpers must land successfully to have their distance recorded and land on sloped ground to prevent serious injury.

A skier begins his descent at point B accelerating down the ramp. At the end of the ramp the skier is travelling horizontally at point E at 32 metres per second (115.2 kilometres per hour).

Let t = the number of seconds in flight after point E $(100, 120)$.
$h(t)$ = the height of the skier above the horizontal ground $y = 0$ (metres)
$x(t)$ = the horizontal position of the skier (metres)

The sloped ground for landing is given by $y = 170 - 0.5x$ where $100 \le x \le 340$.



The ski jumper's suit and skis decrease the horizontal velocity $x'(t)$ so that $x'(t) = 32e^{-0.05t}$.

(a) Show that $x(t) = 740 - 640e^{-0.05t}$. (2 marks)

See next page

It is found that the expression for the position vector for the skier during the flight is given by:

$$\underset{\sim}{r}(t) = \begin{pmatrix} 740 - 640e^{-0.05t} \\ 120 - 2.5t^2 \end{pmatrix}$$

(b) Calculate the height of the skier above the sloped ground after 3 seconds of flight, correct to the nearest 0.01 metre. (3 marks)

(c) Determine the vertical lift s (m/s^2) provided by the skier's suit and equipment in the descent if $\frac{d^2h}{dt^2} = s - 9.8$, where s is a constant. (3 marks)

See next page

Question 19 (continued)

It can be shown that the Cartesian equation for the skier's flight is given by:

$$y = 120 - 1000\left(\ln\left(\frac{740 - x}{640}\right)\right)^2$$

(d) Calculate the time taken for the skier to land on the sloped ground, correct to the nearest 0.01 second. (3 marks)

(e) Calculate the angle at which the skier impacts the sloped ground, correct to the nearest 0.1 degree. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

ACKNOWLEDGEMENTS

Question 19 U.S. Ski & Snowboard. (2018). *Ben Loomis nordic combined 2018 US Olympic team trails at the UOP* [Photograph]. Retrieved April, 2021, from https://usskiandsnowboard.smugmug.com/Jumping-Nordic-Combined/201718-Ski-Jumping-Nordic-Combined/2018-Olympic-Team-Trials-Nordic-Combined/i-ZS4JBFk/A

Published by the School Curriculum and Standards Authority of Western Australia
303 Sevenoaks Street
CANNINGTON WA 6107